Problem of the ‘Week’

Find all solutions to the differential equation

\[ x^2 y'' + 4xy' + 2y = e^x(x^2 + 4x + 2) + 2 \]

which are continuous on the interval \(-1 < x < 1\).

**Solution:** The differential equation is Cauchy-Euler so we solve the auxiliary equation

\[ m(m-1)+4m+2=0 \rightarrow m^2+3m+2=0 \rightarrow (m+2)(m+1)=0 \]

\[ m=-2,-1 \]. This tells us the homogeneous solution

\[ y_h = Ax^{-2} + Bx^{-1} \]. Since the solution must be continuous we have \( A = B = 0 \). Next we see that the particular solution is given by \( y_p = e^x + 1 \). Since this function is continuous we have our solution:

\[ y = e^x + 1 \].

Return solutions to Harris 4151 by 13 February 2015 to be eligible for prize, or submit electronic solutions to jpledford@vcu.edu.