Virginia's Mathematics Specialist Initiative: Preparing Mathematics Specialists

Introduction

Over 20 years of collaboration including a comprehensive project, the Virginia Mathematics Specialist Initiative (VMSI), lead to the development of a program to prepare successful classroom teachers for the K-8 Mathematics Specialist Virginia teaching licensure endorsement and to assume the role of a K-8 school based mathematics specialist. Numerous critical collaborations sustained and moved the VMSI forward. Before describing the core courses that are included in the preparation program a description is shared of the evolution of the initiative and includes information on funding that has provided support for activities in the VMSI, state policy implications and development of a working definition of a school based mathematics specialist. The significant collaborations among institutes of higher education (IHE), school division mathematics leaders, and state organizations dedicated to improving mathematics education are described along with the efforts that created the vision for the knowledge and skills that mathematics specialists need to successfully carry out their various roles. The document concludes by describing the Virginia Mathematics Specialist Program and the core mathematics and leadership courses included in the Program.

Critical Collaborations Leading to the Vision

In 1992 the Virginia Mathematics Coalition, now the Virginia Mathematics and Science Coalition (VMSC), joined with the Virginia Department of Education (VDOE), the Virginia Council of Teachers of Mathematics (VCTM), the Virginia Council for Mathematics Supervision (VCMS) and others in a National Science Foundation (NSF) funded project, V-QUEST, to prepare elementary and middle school classroom teachers to serve as "Math Leaders" or "Science Leaders" in their schools. Over the three years of funding, participating K-8 mathematics and science teachers increased their knowledge in mathematics and science content and in content pedagogy during intensive and focused summer institutes. These classroom based teacher leaders returned to their schools to lead efforts toward improving teaching and learning in mathematics and science.

On May 20, 2002 a significant forum, "Moving from Teacher Leaders to Mathematics Teacher Specialists" took place in Fredericksburg, Virginia engaging representatives from the VMSC, VCMS, VCTM, University mathematicians and mathematics educators, school division leaders, and the VDOE. The forum participants agreed that a well-prepared mathematics teacher specialist could be an effective support for classroom teachers. Funding support for this forum as well as additional forums to gather more information about the work and preparation of mathematics specialist in subsequent years came from the ExxonMobil Education Foundation.

In 2002 the VMSC appointed a task force to investigate how a mathematics specialist embedded in a school might improve instruction and consequently student learning. In addition, the task force was charged with making recommendations for the potential roles and responsibilities a specialist might assume and about the preparation a specialist would need to effectively carry out their work. The Mathematics Specialist Task Force Report (VMSC, 2005) put forward a strong recommendation that *well-prepared mathematics specialists* be placed in

elementary and middle schools to work with teachers to strengthen their mathematics knowledge along with their instructional practices for teaching mathematics so that every Virginia student could reach high levels of mathematics achievement. In addition the Task Force Report made specific recommendations for the necessary knowledge and skills and for the mathematics content and leadership experiences for the coursework.

The Mathematics Specialist Task Force was among the early efforts that came to be identified as the Virginia Mathematics Specialist Initiative (VMSI) under the umbrella of the Virginia Mathematics and Science Coalition (VMSC) which at the time represented over ten years of partnerships. Beginning in summer 2002 the work of developing and offering courses specifically designed to prepare mathematics specialists has been supported with a sequence of three Virginia Mathematics and Science Partnership (MSP) grants involving forty-five Virginia School divisions. In addition the work has been supported with a series of four five-year NSF projects under the VMSC umbrella and guided by the collaboration of Virginia Commonwealth University (VCU), University of Virginia (UVA), Norfolk State University (NSU), Longwood University (LU), and University of Maryland. Research carried out as part of the NSF grant projects confirms the positive benefits of having a well prepared mathematics specialist working with teachers in a school to improve student achievement. This research can be accessed at http://www.vamsc.org/.

- (ESI-0353360) Mathematics Specialist in K-5 Schools: Research and Policy Pilot Study (6/1/04 - 5/31/10)
- (DUE-0412324) MSP Preparing Virginia's Mathematics Specialists (8/1/04-7/31/13 which included several supplements)
- (DRL-0918223) Research the Expansion of K-5 Mathematics Specialist Program into Rural School Systems (9/1/09 – 8/31/15, with the no cost extension)
- (DUE-0926537) MSP Institute: Mathematics Specialists in Middle Schools (8/1/09 – 7/31/15, with the no cost extension)

The VMSI has steered the efforts to use school based mathematics specialists to support teachers in Virginia elementary and then middle schools with three notable successes: 1) The Commonwealth of Virginia has established a mathematics specialist endorsement for elementary and middle education (Virginia Board of Education, 2013, p.66); 2) Twelve state universities have established master's degree programs to prepare mathematics specialists; and 3) The Virginia Board of Education recommended the placement of one specialist in schools for every 1,000 students. Though this recommendation is currently unfunded, school districts have been creative in using state funds such as, Algebra Readiness Funds, and local sources to create specialist positions.

Following the 2002 Task Force, five content courses and three leadership courses where developed and implemented with funds from State MSP grants and the two NSF grants. These courses were focused on development of elementary mathematics specialists. As the first two NSF grants were reaching an end, the research showed that well prepared elementary

mathematics specialists working with teachers were making a difference (Campbell, 2011). However, the VMSI leaders realized there was a missing link between the K-5 preparation Program and the K-8 Mathematics Specialist endorsement. The middle school years needed to be addressed. Consequently, the VMSC formed the 2008 Middle School Mathematics Specialists Task Force to consider how the current Mathematics Specialist Program could be modified to better address the needs of middle school mathematics specialists who would, in turn, better help middle school teachers and students. As with the 2002 Task Force, the participants included university mathematicians, mathematics educators, and district mathematics supervisors. Now, in addition, mathematics specialists could also be invited to participate. The Task Force considered the unique demands relevant to the work of a middle school specialist. The Task Force recognized that the middle school specialist must accommodate students with a wide range of academic needs, have a range of skills to handle scheduling and organizational constraints, find ways to support teachers as they motivate their students who lack confidence or have not been successful for many years, and manage more autonomy in fulfilling the role. Building on the school-based mathematics specialist responsibilities that had been identified by the 2002 Task Force, the 2008 Middle School Task Force presented the following definition of a mathematics specialist and the responsibilities a specialist may be expected to assume in a school.

Recommended School-based Mathematics Specialist Responsibilities

Mathematics Specialists are teacher leaders with strong preparation and background in mathematics content, instructional strategies, and school leadership. Based in elementary and middle schools, Mathematics Specialist are experienced teachers who are released from full-time classroom responsibilities so that they can support the professional growth of their colleagues, promoting enhanced mathematics instruction and student learning throughout their schools, They are responsible for strengthening classroom teachers' understanding of mathematics content, and helping teachers develop more effective mathematics teaching practice that allow all students to reach high standards, as well as sharing research addressing how students learn mathematics.

The overarching purpose of the mathematics specialists is to increase the mathematics achievement of all students in their schools. To do so, they:

- Collaborate with individual teachers, teams of grade level mathematics teachers, and with vertical teams across grade levels through co-planning, co-teaching, and coaching;
- Assist administrative and instructional staff in interpreting data (both formative and summative) and designing approaches to improve student achievement and instruction;
- Collaborate with teachers and teams of teachers to ensure that the school's instructional practices are aligned with state and national standards, as well as their school division's mathematics curriculum;
- Assist teachers' with delivery and understanding of the school curriculum through collaborative long-range and short-range planning;

- Facilitate teachers' use of successful, research-based instructional strategies, including differentiated instruction for diverse learners, and appropriate use of technology;
- Provide job-embedded professional development focused on both mathematical content knowledge and mathematical pedagogy;
- Assist teachers in fostering partnerships with parents/guardians and community leaders to foster continuing home/school/community relationships focused on students' learning of mathematics; and,
- Collaborate with administrators (both in and outside of the mathematics community) to develop a vision and to provide leadership through professional development and for a school-wide mathematics program. (VMSC, 2009. p. 17)

After the task force members completed their work, teams of educators collaborated to add more middle school content to both the content and leadership courses. An additional algebra course was designed for middle school specialists, stronger connections were added between the algebra courses and the numbers and operations and rational numbers courses. The geometry course incorporated high school geometry concepts and use of dynamic geometry software and the probability and statistics course added more analysis of data techniques. Course assignments were revamped to differentiate for participant school level placement and the leadership courses included issues specific to middle school in terms of equity, interdisciplinary teaming, and coteaching. Other challenges such as addressing wider learning gaps and tackling student motivation issues at the middle school level were also incorporated.

Virginia Mathematics Specialist Preparation Program

Twelve Virginia universities currently offer a master's degree program to prepare mathematics specialists each with a unique set of program expectations. However, the universities have modeled their content and leadership coursework on a set of core courses. What follows is a description of the core courses funded through the series of four five-year NSF projects listed above and developed and offered under the VMSC umbrella and guided by the collaboration of Virginia Commonwealth University (VCU), University of Virginia (UVA), Norfolk State University (NSU), Longwood University (LU), and University of Maryland.

Core Mathematics and Leadership Courses

The Mathematics Specialist Program designers realized from the beginning that teaching courses for mathematics specialists would be unique. Instructors would be teaching coaches of mathematics teachers. The program designers could not take a previous college mathematics course in number theory, geometry or algebra and make slight changes to adapt it and meet the needs of mathematics specialists. New courses had to be developed and sequenced into coherent program. Courses were needed that tied mathematics content knowledge to content pedagogical knowledge and allowed teachers to understand the progression of mathematical ideas of their students. In addition these future leaders would need to recognize how the information from their courses tied to their own teaching practices and how it would be reflected in the coaching practices they would implement upon completion of the program. Helping participants recognize

how assignments from their courses translate into their practice both as teachers and as coaches is a critical obligation of the course instructors.

Numbers and Operations (3 graduate mathematics credits)

This introductory course addresses fundamental mathematical ideas concerning the operations of arithmetic and the base-ten number system. Connections between the operations are explored in various contexts including whole numbers, problem solving, decimals and fractions. The structure of the number system is used to develop understandings of our base-ten system. The course also uses cases about students' thinking and the computational methods they use and episodes in the history of the number system that illuminate the developmental progression of the mathematics and the learning trajectories of children.

Rational Numbers and Proportional Reasoning (3 graduate mathematics credits)

In this course students explore the conceptual and procedural basis of rational numbers; fractions, decimals, and percents as well as the essential role that proportional reasoning plays in mathematics. The logic, estimations, interpretations, and procedures used when ordering and computing with fractions and decimals are explored using multiple representations and interpretations including visual and physical representations. Episodes from the history of the number systems are explored and compared with the developmental sequence and learning trajectories of children learning this material.

Functions and Algebra I (3 graduate mathematics credits)

Students develop skills in representation, generalization, and development of mathematical arguments through the exploration of the properties of arithmetic operations, the relationship between operations, and operating on particular numbers. Additional topics from algebra that are explored are: variables, patterns, and functions; modeling and interpretations of graphs; linear functions and non-linear functions, including quadratics and exponentials.

Functions and Algebra II (3 graduate mathematics credits)

This course extends the understanding of topics introduced in the Functions and Algebra I course, introduces new topics from secondary mathematics, and integrates graphing technology into the study of the algebra topics. Class activities focus on extending participants skills in representation, generalization, and developing mathematical arguments. Topics include but are not limited to linear equations and inequalities; modeling and interpreting graphs; linear and non-linear functions; logarithms; factoring, zeros and intercepts; domain and range; exponents and radicals; and some number theory related to the real number system.

Geometry and Measurement (3 graduate mathematics credits)

This course explores the foundations of informal geometry and measurement in 1, 2, and 3 dimensions. The van Hiele model for geometric learning is used as a framework to explore how children build their understandings of length, area, volume, angles, and geometric relationships.

Visualization, spatial reasoning, and geometric modeling are stressed along with transformational geometry, congruence, and similarity.

Probability and Statistics (3 graduate mathematics credits)

Various elementary statistical measures and graphical representations are used to describe, compare, and interpret data sets. The basic laws and concepts of probability are explored including: sample spaces, probability distributions and random variables. A statistical project is required that uses hypothesizing, experimental design, the collection of data, and comparisons of different populations.

Leadership I (3 graduate mathematics education leadership credits)

This introductory course is designed to build an understanding of the content and process standards identified by the National Council of Teachers of Mathematics (NCTM) *Principals and Standards for School Mathematics* (2000) and the *Virginia Mathematics Standards of Learning* in K-8. In addition connections are made with the mathematics content as participants develop their knowledge about mathematics, mathematics content pedagogy, and diagnosing student understanding. A focus is given to students as mathematics learners with attention to learning theory, assessment, and diverse learners; teachers as learners through study groups and observation of another teacher's classroom; and the instructional program through the design, teaching, and evaluation of inquiry based lessons.

Leadership II (3 graduate mathematics education leadership credits)
This course is designed to build skills, understandings and dispositions required for optimal mathematics education leadership roles in K-8 schools with focus on the different roles of the school-based mathematics specialist. Special attention is given to developing coaching skills and working with adult learners, gaining an understanding of mathematics content pedagogy to be able to support teachers, becoming familiar with using research related to topics in selected NCTM strands for instructional decision making, and building deeper understandings of the mathematics that underpins the K-8 mathematics curriculum.

Leadership III (3 graduate mathematics education leadership credits)

This course is designed to build skills, understandings and dispositions required for optimal mathematics education leadership roles in kindergarten through eighth grade with attention given to data analysis, and collaborative data driven discussions for instructional program decision making. Special attention is given to the research on mathematics content and facilitation of the Lesson Study process; on the use of formal and informal formative assessments to guide instruction; the components of effective task-based mathematics instruction; and the components of effective mathematics lesson planning.

Logistics of the Course Offerings

Course Format

As designed a course may be taught as a residential summer institute with 55 hours of class time and significant daily in class and homework assignments including readings, mathematics, and

writing reflection papers. As a traditional semester class a course may be taught in three-hour weekly sessions each with homework assignments including readings, mathematics, writing reflection papers, and writing cases. A third option often used for the leadership courses splits the time between summer sessions and Saturday classes. The timeframe when a course is taught impacts participants experience in different ways. In the summer institute format, students are immersed in the work and have the opportunity for additional collaboration with their peers after class hours. Participants do not, however, have the opportunity to do the mathematics with their own students, to interview students about their understanding of the mathematics, or write their own case studies.

Time can become an issue in any format so careful planning and pacing is essential.

Summer Residential Institute Program Scheduling.

Students participating in a summer institute program take courses each of three consecutive summers. Content courses in a summer institute are offered in intense 2½ week sessions designed for two 3 ½ hour blocks per day. Two content courses are offered in succession over a five week period. In addition, the first half of a leadership course is also scheduled for each summer, held four times spread out over the five week institute. The second half of the leadership course is held on four Saturdays spread throughout the fall semester and meets for 6 hours each time. This allows participants to work with students and teachers in their schools when completing class projects.

Content courses were taught simultaneously during the first summer institute so that on a given day participants experienced one course in the morning and another in the afternoon for the five weeks. Feedback on this schedule was not as positive so the schedule was adjusted to have one course follow the next.

Semester Program Scheduling.

Students are enrolled in a content course each semester which meets one night a week for 3 hours. During the summer the course may meet for two full weeks or for two or three days spread over several weeks. Leadership courses are generally offered in the fall semester sometimes overlapping into the second semester to allow participants more time to work with students and teachers in completing class projects.

Blended Learning Format

The core courses of the Mathematics Specialist Program were originally designed to be face to face courses but in reaching out to rural school districts preparing elementary mathematics specialists it became evident that travel to class would be a major obstacle to teacher participation. To address this challenge the program was offered in a combination of course formats; blended and summer residential institutes.

Technology has allowed the courses to be repurposed to fit a blended format. To maintain the cohesiveness of the cohort the blended courses meet twice for a 2-day Friday and Saturday face to face meeting, at the beginning and then midway through the course, with the remainder of the

classes meeting synchronously online. The sequence of courses remains similar to the original design with a few exceptions. Courses begin with the *Number and Operations* course. The online platform allows chat rooms for small group conversations and group work which instructors can visit, whole group real time discussions, and group projects are assigned with the expectation that teachers will meet with their groups online.

Each summer there is a two week face to face course. The first summer *is Geometry and Measurement*, the second summer is *Algebra and Functions I* and the third summer is *Mathematics for Diverse Populations*. Another summer course is taught in the blended format each of the next two summers; first is *Rational Numbers and Proportional Reasoning* and then *Probability and Statistics*. All three Leadership courses are taught in a blended format during the school year. *Leadership I* and *II* are taught the fall and spring semester following the first and second summers respectively and *Leadership III* follows the third summer institute during the fall semester.

Course Sequencing

The recommended sequence of content courses shown in Table I begins with the *Number and Operations* course because it provides the foundation for the philosophy and pedagogical methodology of the Program. This is followed by the *Rational Number and Proportional Reasoning* course. In a summer institute program these are offered the first summer over lapped by the *Leadership I* course which finishes at the end of the fall semester. Under a semester schedule, *Number and Operations* is in the fall semester and the *Rational Number* course is in the spring. *Leadership I* begins in September but spreads into the second semester ending in January or February.

Table 1. **Table of Core Course Sequencing**

Mathematics Content Courses for Mathematics Specialists	**Mathematics Education Leadership Courses for Mathematics Specialists**
Number and Operations (Required first course)	Leadership I
Rational Number and Proportional Reasoning	
Geometry and Measurement	Leadership II
Algebra and Functions	
Probability and Statistics	Leadership III
Algebra for Middle School Math Specialists	

The second year of a summer institute format includes the *Geometry and Measurement* course and the *Algebra and Functions I* course along with the first half of *Leadership II* which

concludes at the end of the fall semester. In a semester schedule, *Geometry and Measurement* is offered in the fall, followed by the *Algebra and Functions* I course in the spring, with the *Leadership II c*ourse running from September through February.

The third year a summer institute program involves *Probability and Statistics* followed by Functions and *Algebra II for Middle School Mathematics Specialists* or a diverse learners course is offered if only elementary specialists are involved. *Leadership III* begins in the summer and concludes at the end of the fall semester. In the semester schedule, *Probability and Statistics* is offered in the fall along with the *Leadership III* course, and then *Functions and Algebra II* for middle school mathematics specialists in the spring. The *Leadership III* course concludes in the end of the fall semester even in the semester format to allow students to concentrate on the particular university requirements for their practicum or externship commitments during the last semester of their program.

Instructional Teams and Coteaching

Because of the unique preparation necessary to assume the role of a mathematics specialist the specialist courses are taught by instructor teams usually comprised of three instructors with different backgrounds and experiences as mathematics educators. For the content courses it is critical to have a higher education mathematician on the team as the content courses carry graduate credit in mathematics and though course content is focused on K-8 mathematical concepts understanding of the deeper mathematics connections is critical. The mathematics instructor is joined by a university mathematics educator or a K-12 mathematics supervisor with adjunct experience and an experienced mathematics specialist. The teaching team works collaboratively to analyze the mathematics and to also connect the mathematics to teacher practice. Planning, teaching and assessment are all cooperatively done as each instructor models coteaching and what research says about good teaching practices.

The leadership courses carry mathematics education graduate credit so it is critical that a mathematics education instructor or adjunct instructor from an institute of higher learning be on the team. Other members of this team should include a K-12 mathematics supervisor and an experienced mathematics specialist. Participants will be analyzing their own teaching, learning to coach one-on-one and in small groups, and learning how to best impact the mathematics program in their school. It is critical that the instructors have experience in a K-12 school setting working with teachers and administrators to support a strong mathematics program. Each member of the teaching team brings an expertise to the team and must contribute collaboratively to the planning, teaching and assessment of the students in the class. The instructors and the students are all professionals with much to offer to the learning environment and much to learn from each other.

Prior to a summer institute, the program leaders have provided opportunity for the course instructors to have a day-long meeting to set goals, review any data from the outside evaluators and from student observations and course exit slips, and coordinate their work. Instructors again touch base as students transition from one summer course to the next discussing potential connections that can be made among courses as well as needs and challenges. It is particularly helpful for the leadership course instructors to hear from the content course instructors to plan for projects in the leadership courses. Numerous instructors have been involved in more than one

course during a program and this continuity has been helpful. It is also beneficial to bring in new instructors to work with veteran individuals to build the instructional team and allow the students to experience the teaching styles and knowledge others bring to the work.

Cohort

When possible courses should be offered to a group of teachers who are committed to completing the work required for the entire Mathematics Specialist Program. The support that teachers can offer one another in a cohort is critical, especially as the course work increases in content difficulty. The courses are designed to include, productive math talk discussions, group project based learning opportunities, and collaborative in class group work. Therefore, a cohort provides an opportunity for the students to develop a bond with one another as they continue taking courses and share more and more of their mathematical and pedagogical thinking. This bond extends their networking with each other beyond the program as the teachers assume roles of mathematics specialists.

Instructors must be sensitive to the demands on the students who are trying to juggle their course work and full time teaching responsibilities as well as family needs. Instructors from one course should be in communication with instructors of previous courses in the sequence so they are fully versed in the prior knowledge held by the group. If all students have participated in the same cohort this background knowledge can be more accurately defined. Program leaders provided opportunity to bring instructors together to review course content and check on student progress.

Regardless of the scheduling or formatting of courses the Virginia Mathematics Specialist Program remains focused on providing the foundation strong teacher leaders need to successfully transition from the classroom to becoming mathematics specialists.

Teaching Coaches of Mathematics Teachers

The instructors in the Virginia Mathematics Specialist Program strive to prepare strong classroom teachers with at least three years of teaching experience with the knowledge, skills, and dispositions that are particular to the leadership they must provide in their new role as mathematics specialists and coaches of teachers. The VMSC Task Force definition of mathematics specialists (VMSC, 2009. p. 17) sets the goal for what these teachers should know and be able to do. Moving from being a successful classroom teacher to being a successful coach is a challenging task. Mathematics specialists are coaches for their fellow teachers, supporting them in more effective planning, teaching and assessing of their students' mathematical conceptual knowledge and procedural skills.. Much thought and research went into the development of each course as well as the careful fusion of these courses into a program that will provide the best cohesive path for preparing successful teachers to become successful coaches. Instructors are intentional about considering what it means to teach coaches of teachers as they facilitate the learning in order to develop mathematics specialists' leadership skills.

In the 2002 Virginia Mathematics and Science Coalition (VMSC) Task Force Report recommendations were made about how mathematics specialists should be prepared for the leadership roles they assume in schools (VMSC, 2005).

> To build leadership skills, courses must be offered that will enable candidates to build a deep understanding of how students learn mathematics and of pedagogical knowledge specific to mathematics teaching and learning. Candidates will learn to develop curriculum that is based on current research, including national and state standards for mathematics and will design instruction that meets the needs of diverse learners.
>
> Course work will enable candidates for the Mathematics Specialist endorsement to develop skills in analyzing individual student performance on a variety of assessment protocols, and in analyzing and interpreting individual as well as collective test data. They will use the results from these analyses to inform instructional decisions. In addition, candidates will learn to gather and interpret relevant data about instructional strategies and instructional programs to facilitate improvements in student learning. (VMSC, 2005, p. 18-19)

In addition, because the role of the mathematics specialist is new to many principals, teachers, parents and students careful attention must be given to helping participants learn to work as a collaborative leader with all school stakeholders. Participants must develop their communication skills and ability to work with adult learners in order to help build a strong and effective mathematics program within their school Finding successful ways to identify needs, communicating these needs in a tactful and positive manner and working collaboratively to implement steps to meet these needs, are leadership skills each mathematics specialist must be allowed to develop during the program. To help meet these expectations all courses in the program are designed to actively engage the participants in group work and in simulations of leading activities in class and in their schools.

Modeling Best Practices

Instructors are purposeful about modeling leadership and instruction that reflects collaboration and best practices. Instructional strategies are designed to build a community of learners among the participants, just as instructors hope the participants will strive to build within their schools. The focus throughout the program is always on the mathematics and course instructors bring attention to the mathematics content, the developmental progression of the mathematical concepts, how children make sense of the mathematics, and which pedagogical moves afford students opportunities to become better mathematical thinkers. Small group and whole group discussions are grounded in classroom practice using written and video cases of mathematics, cooperative group work is focused on mathematics content and mathematics content pedagogy, and writing assignments require participants to reflect transferring ideas from class discussions and projects into their practice. Teachers and instructors function as colleagues sharing knowledge gained from their diverse practices and experiences which models how teachers and a mathematics specialist work together.

Instructors also model good questioning techniques and use formative assessment to more effectively help participants construct their understanding of the mathematics and to inform the instructors about the connections participants make among the courses that deepen their understanding of the mathematics content strengthening their ability to recognize and articulate how children construct mathematical understandings. While mathematics is the focus for the

content courses, it is also important, to include case studies as part of the participants' experiences as a venue to deepen their understanding of how children make sense of the mathematics. The case studies bring validity to a more flexible way of thinking about mathematics. Participants do need to attain a certain level of understanding about the mathematics to enter into a deeper conversation about the mathematics in the cases. Consequently, attention should be given to developing the mathematical understanding of the participants to connect the mathematics back to the cases. There are occasions when the instructor needs to combine the case study discussion with the mathematics discussions.

Supporting inquiry-based learning is modeled in all courses through projects, tasks, and class discussions. Carefully constructed assignments allow participants to develop and communicate their mathematical and pedagogical understanding. A problem may involve solving a mathematical task in a collaborative group or just turning and talking to a shoulder partner about using a different strategy. In leadership courses participants conduct student interviews, work one-on-one with a mathematics teacher, meet with their principal and examine school data to determine school needs and plan professional development. A culminating project in *Leadership III* is a lesson study project in which a team of teachers is responsible for writing, implementing and assessing a detailed inquiry based lesson. This experience provides participants with the opportunity to reflect on their strengths and weaknesses as a communicator and collaborator in working with a team to impact student learning.

Lesson Learned

It became clear early in the program that participants needed to be challenged more with writing assignments. As participants progress through the program they are challenged with writing assignments both in the mathematics and the leadership courses. The instructors have recognized that writing, as a reflection of one's analytical and communication skills is of critical importance to a mathematics specialist. As a coach, they will be required to summarize research findings for their principal and staff, share data reports, collaborate to develop a school vision for the mathematics program, assist a teacher in developing a more engaging lesson, plan professional development that meets the needs of teachers with diverse experiences, work with a grade level team that is reluctant to share, talk with parents that do not understand an inquiry based curriculum and learn to celebrate the "baby steps" that help move the mathematics program forward. As mathematics specialists improves their writing skills, other communication skills are strengthened and their credibility is enhanced. Not only are the topics of the writing tasks important, the practice of writing is valuable to the future work of a specialist.

In addition to modeling good teaching practices, the instructors realized the need to make those practices explicit. In order to do this the instructors "stepped in and out" of their role as facilitator to talk specifically about facilitation moves. This can generate a meaningful discussion about how the instructor designed the task; or why they decided to ask one student to share their idea, prior to another; or why they had students chart their ideas; or decided on a certain focus question; or why the groups were changed for a particular activity. Being explicit about teaching moves, and coaching moves, points out the complexity and magnitude of the decisions that mathematics specialist must make every day. The goal being that as participants

more carefully plan and anticipate their teaching and coaching moves, they will become increasingly more effective in their role as coaches of teachers.

The 20 year collaboration between the members of the Virginia Mathematics and Science Coalition, the Virginia Department of Education, K-12 mathematics leaders, as well as, mathematicians and mathematics educators in institutes of higher education has led to a comprehensive program to prepare mathematics specialists. The research, planning, coordination and effective evaluation of the Program have been exemplary. Research is still being conducted on the impact of mathematics specialists in K-8 schools but the current Program is making a positive difference in preparing coaches of teachers.

Enhanced Syllabi for Two Core Courses

The core mathematics and leadership courses to prepare successful classroom teachers for the K-8 Mathematics Virginia teaching licensure endorsement and the role of a K-8 mathematics specialist have been developed and refined overtime as a critical component of the Virginia Mathematics Specialist Initiative (VMSI). As a result of the grant supported projects over the past eight years much feedback on impact of each course on participants knowledge and growth in mathematical pedogogy has been gathered from outside evaluators. This feedback as well as the artifacts from instructors after they have taught the courses has been used to compile an enhanced syllabus for each of the core courses. The enhanced syllabi provide future instructors with the general structure and expectations for the core courses in the mathematics specialist program. In additon the syllabi provide specific information in the form of a course outline with topics and essential questions and a sample of a lesson plan. In addition, enhanced syllabi for each course describe the course goals, course overview, course format and key activities, and course materials (primary student texts, instructor primary resources, and supplementary readings).

Attached are samples of course syllabi for two of the core courses*: Number and Operations for Mathematics Specialists* course is included as Appendix A and the enhanced syllabus for *Leadership II for Mathematics Specialists* course is included as Appendix B.

Appendix A

Numbers and Operations for Mathematics Specialist

Numbers and Operations is a 3-credit hour graduate mathematics course designed to contribute to preparing teachers with at least three years of classroom teaching experience to become school-based mathematics specialists. This is the first graduate credit mathematics course in the program for future K-8 mathematics specialists. The course focuses on the number and standard described in the National Council of Teachers of Mathematics *(NCTM) Principals and Standards for School Mathematics* (2000) and the number and number sense and computation strands described in the *Virginia's Standards of Learning in Mathematics Curriculum Framework (VDOE,2009).* The course develops a comprehensive understanding of the base ten number system, its structure, the role this structure plays in problem solving and computations, and the properties of arithmetic that form the foundation for algebra. Attention is given to connecting these mathematics concepts to school students' thinking as they solve problems and construct their understanding of the number system and develop their proficiency in arithmetic computation.

Course Goals

The goals of the *Number and Operations* course is to engage students to construct a deeper conceptual understanding of the base ten number system, to identify the relationships among the four operations, and to understand the mathematics that underpins different computational strategies for whole numbers and decimals. In addition students develop an understanding of fractions as numbers and how that understanding supports comparing, ordering, and operating on positive fractions. Students also explore the role that multiple representations play in developing mathematical understanding and presenting mathematical arguments. This course will:

1. Develop an understanding of the structure of the base ten number system that influences learning to count, performing operations with multi-digit numbers, and working with decimal numbers.
2. Develop the knowledge and skill to represent and interpret quantitative situations verbally, pictorially, and symbolically.
3. Investigate a variety of situations modeled by the four basic arithmetic operations; addition, subtraction, multiplication, and division, and examine various representations of the four operations.

4. Develop the knowledge and skills to recognize generalizations in different computational situations that lead understanding and representing the properties that support the operations verbally and symbolically.
5. Develop an understanding of positive fractions as numbers and how understanding of the principals that govern whole number operations need to be expanded to operate with positive fractions.
6. Study the mathematical progression of concepts and how children develop their understanding of number and operations using case studies.
7. Investigate mathematics ideas about number and operations by engaging in problem solving, reasoning, conjecture, and developing and defending mathematical arguments.

Course Overview

The structure and logic of the base ten number system is fundamental to understanding and reasoning about the four arithmetic operations and developing computational fluency. Often elementary and middle school mathematics teachers have a limited understanding of the structure of the Hindu-Arabic numeral system. That is, an understanding of the supporting structure and logic of a positional base ten number system. In addition, teachers need to extend their understanding of quantity represented by whole numbers and develop their conceptual understanding for quantity represented by rational numbers. During the course, teachers make connections among the four operations for whole numbers and positive rational numbers. In summary, the purpose of this course is to provide opportunities for prospective mathematics specialists to build their knowledge, skills, and mathematical dispositions to support teacher and student learning in number and operations.

Instruction for the *Numbers and Operations* course relies heavily on two of the Developing Mathematical Ideas (DMI) modules which include rich mathematical tasks and case studies and then supplemented with additional readings, mathematics activities, and analyzing school classroom artifacts. The course begins with exploring the base ten number system structure, considering how the structure supports multi-digit computational procedures, and examining how basic concepts of whole numbers reappear when the set of numbers is expanded to include working with decimals. The first half of the course the uses DMI *Building a System of Ten (BST)* as the primary text. The second half of the course moves to the DMI module, *Making Meaning of Operations* (MMO). In the last half of the course participants examine the different actions and situations modeled by addition, subtraction, multiplication, and division in order to make meaning of each of the operations and the relationships among the operations. Using the print and video cases in MMO students first examine young children's counting strategies as they encounter word problems and then move to an examination of the four basic operations on whole numbers. Students then revisit the operations in the context of positive fractions.

Throughout the course participants engage in making conjectures, developing generalizations, and making mathematical arguments. Through classroom discussions and identifying

generalizations in arithmetic, students build a working knowledge of the properties of arithmetic that will be formalized in the *Functions and Algebra* course for mathematics specialist. Decimal numbers are given a thorough treatment during the *Numbers and Operations* course. Participants spend some time in *Numbers and Operations* making sense out of fractions as numbers and the role that fractions play in quantifying different relationships. The principles governing computation with whole numbers are re-examined in light of the four basic operations with positive fractions. In the *Functions and Algebra* course students will expand their understanding of real numbers to include integers and computation with integers.. In the *Rational Number and Proportional Reasoning* course students engage in a more thorough study of fractions.

Instructors should be aware that the ideas introduced in this course will be examined again through an algebra lens in the *Functions Algebra* course. For example, as participants in that course look at generalizations and then laws of arithmetic, they will revisit where notions of these laws began to develop in the *Numbers and Operations* course. For instance, the Distributive Property is used by students when they create array models for multiplication, with both whole numbers and rational numbers: do participants recognize 6 ½ x 4 = 6 x 4 + ½ x 4 as the distributive property? In other words, do they see it as more than the property, but as a strategy to help with computation?

Course Format and Key Activities

A variety of formats has been employed to teach The *Numbers and Operations* course. For example, it has been taught as two-week residential and commuter summer institutes with 54 hours of class time and significant in class work and homework assignments and as a school-year course with 15 three-hour sessions in one semester or spread over two semesters. The timeframe for teaching the course can impact participants experience and the instructors will need to plan accordingly. One benefit of the summer institutes is that students immerse themselves in the course and have the opportunity for additional collaboration with their peers and the instructors after class hours. This is particularly true for residential institutes. However, in the summer there are few if any relevant opportunities readily available do the mathematics with their students during the class; to interview students about their understanding of the mathematics or write cases based on their students. Instructors in the summer institutes use video of student interviews and bring samples of student work from other sources to provide an opportunity for students to experience analyzing where students are in the developmental progression for number concepts and skills and to make recommendations for next instructional steps

Instructional methodology includes small group and whole group discussions anchored in written and video cases of student's mathematics thinking; cooperative group work around mathematics content and mathematics content pedagogy; and analyzing student interviews, student work, and cases from participants' practice. While developing the participant's mathematics content knowledge for teaching is the focus for the course it is just as important to include the case

studies as a venue to deepen their understanding of how children make sense of the mathematics. The case studies also bring validity to teaching mathematics for understanding as well as for computational fluency. Class discussions about the mathematics, as well as the cases, become more robust as participants develop deeper understandings of the mathematics and the developmental progression for various mathematics topics. Course instructors bring explicit attention to the mathematics content, the developmental progression of the mathematics concepts, how children make sense of the mathematics, and which pedagogical moves afford students opportunities to become mathematical thinkers. Instructors intentionally model inquiry teaching for the participants.

Ongoing informal and formal formative assessment is an important component of the course. Based on what they learn about students understanding, instructors continually adapt the class activities and course projects to support participant growth as they construct their understanding and make connections to their classroom practice that deepen their understanding of how students make sense of the mathematics. The course projects include maintaining a portfolio of the mathematics problem sets assigned for homework, completing two student interviews, analyzing two sets of student work, and maintaining a reflection journal throughout the course. Instructors provide guidelines and rubrics specific to each project: and writing is an important component of each project. In addition to the ongoing informal and formal formative assessments there are three summative assessments; a midterm assessment, a cumulative final exam, and final reflection synthesis paper.

Course Materials

Listed below are the primary student and instructor texts for the course. In addition, instructors include supplementary readings from sources such as NCTM journal articles.

The Primary Student Texts

Schifter, D., Bastable, V., & Russell, S.J. (2010). *Developing mathematical ideas, Number and operations, part 1: Building a system of tens, Calculating with whole numbers and decimals. Casebook.* Upper Saddle River, NJ: Pearson.

Schifter, D., Bastable, V., & Russell, S.J. (2010). *Developing mathematical ideas, Number and operations, part 2, Making meaning for operations in the domains of whole numbers and fractions. Casebook.* Upper Saddle River, NJ: Pearson.

Instructor Primary Resources

Schifter, D., Bastable, V., & Russell, S.J. (2010). *Developing mathematical ideas, Number and operations, part 1: Building a system of tens, Calculating with whole numbers and decimal. Facilitator's guide and video*. Upper Saddle River, NJ: Pearson.

Schifter, D., Bastable, V., & Russell, S.J. (2010). *Developing mathematical ideas, Number and operations, part 2, Making meaning for operations in the domains of whole numbers and fractions. Facilitator's guide and video*. Upper Saddle River, NJ: Pearson.

Instructors can find additional information from the developers and various implementers of the DMI materials at http://www2.edc.org/cdt/dmi/dmiless.html.

Instructor Supplementary Resources

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Cramer, K., Behr, M., Post, T., & Lesh, R. (1997). *Rational number project fraction lessons for the middle grades level 1*. Retrieved http://www.education.umn.edu/rationalnumberproject/97_2/97_2.html

Cramer, K., Behr, M., Post, T., & Lesh, R. (1997). *Rational number project fraction lessons for the middle grades level 2*. Retrieved http://education.umn.edu/rationalnumberproject/rnp2.html

Fosnot, C. T., & Dolk, M. (2001a). *Young mathematicians at work: Constructing number sense, addition, and subtraction*. Portsmouth, NH: Heinemann.

Fosnot, C. T., & Dolk, M. (2001b). *Young mathematicians at work: Constructing multiplication and division*. Portsmouth, NH: Heinemann.

Fosnot, C. T., & Dolk, M. (2001c). *Young mathematicians at work: Constructing fractions, decimals, and percents*. Portsmouth, NH: Heinemann.

Kamii,C. (2000) Young children invent arithmetic: Implications of Piaget. New York, N.Y., Teachers College Press.

Kilpatrick, J., Swafford, J., and Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Parrish, S D. 2010. *Number talks: Helping children build mental math and computations strategies.* Sausalito, CA: Math Solutions.

Richardson, K. (2012). *How children learn number concepts: A guide to the critical learning phases.* Bellingham, WA: Math Perspectives Teacher Development Center.

Wright, R.J., Martland, J. & Stafford, A. K. (2000*) Early numeracy: assessment for teaching and intervention.* Thousand Oaks, CA: Sage Publications.

Supplementary Readings for Students

Behrend, J. (2003). Learning disabled students make sense of mathematics. *Teaching Children Mathematics, 9(*5), 269-273.

Clark, D., Roche, A., Mitchell, A. (2008). ten practical tips for making fractions come alive and make sense. *Mathematics Teaching in the Middle Sch*ool, *13(*7), 372-379.

Kaplan, R.G., King, B, Dickens, N., Stanley, V. (2000). Teacher-clinicians encourage children to think as mathematicians. *Teaching Children Mathematics, 6(*6), 406-411.

Parrish, S. D. (2011). Number talks build numerical reasoning. *Teaching Children Mathematics. 13*(3), 198-206.

Pickreign, J. & Rogers, R. (2006). *Do you understand your algorithms? Mathematics Teaching in the Middle Sch*ool, *12(*1), 42-47.

Riddle, M. (2000). Fractions: What happens between kindergarten and the army? *Teaching Children Mathematics, 7(*4), 20-206.

Russell, S.J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics.* 154-158.

Torrence, E. (2003). Learning to think: An American third grader discovers mathematics in Holland. *Teaching Children Mathematics, 10(*2), 90-93.

Wilson, P. (2001). Zero: A special case. *Mathematics Teaching in the Middle School. 6(*5), 300-303.

Numbers and Operations Course Outline: Topics and Essential Questions

The overview displayed in Table 1 presents the scope and sequence of the *Numbers and Operations* course taught in 15 weekly 3-hour sessions. Two 3-hour classes are merged for one full day summer class. The overview identifies the topics and essential questions for each class and resources used to support each class. The *Numbers and Operations* course relies heavily on the mathematics activities and the print and video case studies included in the two Developing Mathematical Ideas (DMI) modules. Instructors supplement and extend the mathematics through additional mathematics activities and readings. The course textbooks are noted in the outline as follows.

- **BST** indicates the *Developing mathematical ideas, Number and operations, part 1: Building a system of tens, Calculating with whole numbers and decimals. Casebook.*

- **MMO** indicates the *Developing mathematical ideas, Number and operations, part 2*, Making *Meaning for operations in the domains of whole numbers and fractions. Casebook.*

Table 1. Numbers and Operations Course Overview

Class	Topics/Resources	Essential Questions
1	Operation of addition Traditional and alternative addition strategies Number talks and mental math as an instructional tool Commutative and associative property of addition BST Chp 1	How do the base ten structure of the number system and the properties of the operations shape the strategies for multi-digit computation? In what ways can numbers be composed and decomposed? How do different visual or physical representations of number highlight the tens structure of the number system?
2	Base ten structure Magnitude Powers of 10 Place value Traditional and alternative subtraction strategies History of the Hindu-Arabic numeration system Activity: Xmania http://mathinscience.info/teach/612_math/math68/count_on_it/xmania_backup/xmania.htm BST Chp 2	What is the structure of the base 10 or Hindu-Arabic numeration system? What are the keys to understanding any numeration system? What is the role of unitizing in developing understanding of place value? What happens when a number is multiplied by power of 10 such as 10, 100, 1000 and so forth?
3	Operation of subtraction Models for subtraction Generalizations: a+b = (a+c) + (b-c) a-b = (a+c) - (b+c) a+0=a a-a=0 BST Chp 3	How do computational strategies for multidigit addition and subtraction rely on the place value and the structure of the base 10 number system? How do different representations illustrate quantity and how are different representations related to each other? How do different representations illustrate the operations of addition and subtraction and how are different representations related to each other?

	Sample lesson for class 3 follows the course overview.	Why is understanding zero challenging for students? What does it mean to be fluent with computation?
4	Operation of multiplication Traditional and alternative strategies for multiplication Models for multiplication Commutative and associative property of multiplication Distributive property BST Chp 4	What is the relationship between multiplication and addition? How do the procedures for calculating a multidigit multiplication problem rely on place value and the structure of the base 10 number system? How do different representations illustrate multiplication and how are different representations related to each other? What role does the distributive property play in multiplication?
5	Operation of division Traditional and alternative strategies for division Partitive division Quotative division BST Chp 5	How do the procedures for calculating a multidigit division problem rely on the base 10 structure of the number system? How do the strategies for decomposing numbers work or not work for division? What models for representing and thinking about partitive and quotative division support student understanding of division? How can using and analyzing different representations support students' understanding of multiplication and division? How do different contextual situations lead to different models for division?
6	Place value representations of numbers less than1 Comparing and ordering decimal numbers Addition with decimals Subtraction with decimals BST Chp 6	How do students use what they understand about whole numbers and place value when they begin working with decimals? What new ideas do students need in order to understand what decimals are and the role they play in representing quantity? How do different representations illustrate decimals and

		how are different representations related to each other? Why do the same principles that govern whole number addition and subtraction apply to addition and subtraction of numbers involving decimals?
7	Multiplication with decimals Division with decimals BST Chp 7 Take home Midterm Exam	How do the same principles that govern whole number multiplication and division apply to multiplying and dividing numbers involving decimals?
8	Addition and subtraction as inverse operations If $a + b = c$ then $a = c - b$ and $b = c - a$ Identity element for addition Number line as a tool to represent the different models for subtraction Classification of addition and subtraction word problems MMO Chp 1	What do children understand when they use counting strategies to solve problems before they learn to add and subtract? Why can the same situation be represented by an addition and subtraction sentence? How can the inverse relationship between addition and subtract be developed using a number line? How do different contextual situations lead to different models or interpretations for subtraction? How can different representations support students' understanding of addition and subtraction?
9	Multiplicative reasoning Additive reasoning Multiplication and division as inverse operations If $a \times b = c$ then $a = c \div b$ and $b = c \div a$ Identity element for multiplication 0 as a factor, as a divisor, and as a dividend MMO Chp 2	What does it mean to understand the *concept* of addition, subtraction, multiplication, and division? How can examining different number sentences that can model a single situation develop an understanding about the relationships between operations? What does it mean to reason additively? Multiplicatively? How do different contextual situations lead to different models for division? How does dividing a whole into parts lead to thinking about fractions as numbers? What role does context play when interpreting the remainder in division? What relationships exist among the four basic arithmetic operations?

10	Rational numbers Partitioning Iterating Unit fraction Density of fractions Area, set, and measurement model for representing fractions Fractions as numbers representing different relationships MMO Chp 3	How do the area, set, and measurement models for fractions highlight different ways of thinking about fractions as numbers? How do different contexts support the five main interpretations: fractions as parts of wholes; fractions as the result of dividing two numbers; fractions as the ratio of two quantities; fractions as operators; and fractions as measures? What is the role of the numerator and the denominator in a fraction? How can partitioning a whole and iterating to create a whole support understanding fractions?
12	Unit fractions Equivalent fractions Comparing and ordering fractions MMO Chp 4 The operations of addition and subtraction with fractions Common units MMO Chp 5	Why is knowing the unit or whole when working with fractions necessary? What does it mean to have a unitary view of fractions? What does it mean when two fractions are equivalent? Why can different fractions name the same point on the number line? Why can different fractions name the same area of a region? What reasoning strategies support students in efficiently comparing and ordering fractions? Why does changing the unit result in different fractional names for the same quantity? What does it mean to have "fraction sense"?
13	The operations of multiplication and division with fractions Inverse property for multiplication MMO Chp 6	How does the meaning of multiplication and division need to be extended when the set of numbers operated on is expanded to include both whole numbers and positive fractions? How do different verbal, visual, and physical representations of fractions highlight the different

		computation strategies for multiplication and division with fractions? What concepts support understanding the invert and multiply algorithm for division of fractions? What concepts support understanding the common denominator algorithm for division of fractions?
14	Division with fractions MMO Chp 7 Review for the final exam MMO Chp 8	What role does the unit play in making sense of the remainder in division with fractions? How are the partitive and quotative types of division seen in problems with fractions? What does it mean to model a situation with an arithmetic sentence or expression? What are the four categories that support context that involve multiplication and division with positive fractions? (MMO FG p. 237) How are common fractions, decimals and percents alike and different?
15	Final Exam	

Sample Lesson Plan for Class 3, Numbers and Operations Course

- **Textbook:** BST Chapter 3 Making Sense of Addition and Subtraction Algorithms
- **Materials:** base 10 blocks, interlocking cubes, HO BST Chp 3 Focus Questions p. 115, HO BST FG Math Activity p. 114, HO with guidelines for the Student Work Samples Writing Assignment.

Essential Questions

How do computational strategies for multidigit addition and subtraction rely on the place value and the structure of the base 10 number system?

How do different representations illustrate quantity and how are different representations related to each other?

How do different representations illustrate the operations of addition and subtraction and how are different representations related to each other?

Why is understanding zero challenging for students?

What does it mean to be fluent with computation?

Math Talk Warm Up (20 minutes)

Present the expressions, 128 + 85 and 63 - 25, one at a time and spend about 10 minutes per problem.

- Individually: Find two mathematically different ways to solve each problem mentally.
- Table groups: Discuss the mathematics behind the different methods used in the table group. Instructor circulates and picks out several different methods to share whole group.
- Whole group: Pick several participants to share their one of their methods. Then ask for whole group discussion about how the different ways are mathematically different.

Link Back to Class 2 and Xmania (20 minutes)

Have a handout or write the following on the board and allow 10 minutes for students to work and 10 minutes to share.

- When working in base 4 what numerical symbols or digits would be used to represent the quantity?
- Draw the base 4 blocks or pieces to represent the base 10 number 29.
- In base 4 what place value is represented by each position or place in the number ___ ___ ___ ___

Math Activity Addition and Subtraction Strategies (45 minutes) Whole group introduction: Need handout Math Activity: Addition and Subtraction Strategies from BST FG Chp 3 p. 114.Complete question #1 together. Prepare participants to develop a poster for 1d by asking them to analyze and describe each student's work displayed in 1a, 1 b, and 1 c. The descriptions should focus on the mathematics and properties that support the work. After the discussion work a class to develop a poster for 1d which will include a verbal description of the problem, a pictorial representation that models the strategy, and a story context that matches the actions in the story. When the class poster is complete, discuss the mathematics and any generalization that may lead to a property, the questions below can be used if needed.

1) What does this work tell about addition?
2) How do we maintain equivalence if we change the addends or the problem? What do we learn about equivalence and addition from this work?

3) If participants are only able to describe the generalization in words lead them to the symbolic representation, (a + b) = (a - c) + (b - c).

- Small group: Assign each table group one of the student subtraction strategies in #2 to analyze. They should make posters similar to the one made in the whole group for #1 regarding addition.
- Whole group: Each will display their poster for a gallery walk.

Posters Gallery Walk and Math Discussion (20 minutes)

- While viewing posters consider the following questions:
  1) What mathematics is evident in these procedures?
  2) What do students need to understand in order to compute in these ways?
- Whole Group Math Discussion, addition and subtraction strategies. First, make sure 2 b,c, and e are clear.
  1) How is 2b the same or different from 1d?
  2) What do you notice when you examine 2c?
  3) What comments do you have about 2e?
  4) What generalizations did you note for any of the subtraction strategies?
  5) What similarities and differences did you note between the addition and subtraction strategies?

DVD for BST Chp 3 (10 minutes)

If time is short during this class use the DVD at the beginning of class 4 as a link back to class 3.

- The DVD will be watched straight through; any points the participants want to discuss should be written down and brought forward in case discussion. Ask them to think about the following questions as they watch the DVD and make notes of specific evidence from what the children are doing and/or saying.
  1) How is an understanding of place value necessary for an understanding of addition and subtraction?
  2) How does the realization of a generalization that leads to understanding a property develop as children's addition and subtraction strategies develop?

BST Chapter 3 Case Discussion (30 minutes)

Many of the ideas addressed in the focus questions are brought out in the Warm Up and the Mathematics Activity so most of the time can be used to bring forth summary ideas.

- Small group discussion will be guided by the Focus Questions for BST CHP 3.
- Whole group discussion is guided by the following questions.

1) How are the mathematics strategies we see students create based on the same underlying principles as the standard algorithms?
2) What are those principles?
3) How does the development of these understandings serve as early development of algebraic thinking and the understanding of algebraic properties?

Reading and Discussion, What does it mean to be computational fluent? (20 minutes)

- Read the short article, Russell, S.J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics.* Then, write a "matchbook" definition computational fluency that you can use when talking with teachers and parents. If there is not sufficient time in class, this can be assigned for homework.
- Small Groups will share their definition and pick one to share with the whole class.
- Whole Group: Have small groups share and then pose the question, as you compare each definition what was the key idea(s) that emerged.

Exit Card Prompt:

How do you think the standard algorithm best fits into the curriculum and/or instruction?

Homework for Class 4

1. Writing Assignment: Analyzing subtraction algorithms. Provide HO BST FG p. 116. This will be due next class for class discussion and then will be put into the portfolio that is due in Class 5.

2. Writing Assignment: Students' Work Samples. We will share in groups next week, and you will turn in your paper at the end of class..

In this course, we will explore the ways students engage with the topics of the elementary and middle school mathematics curriculum. Part of our next session will be devoted to discussion of the mathematical goals we have for our students. In preparation for this discussion, please complete the following assignment.

Ask your students a question (give them a problem) relating to the multi-digit computation. The problem can involve addition or subtraction depending on the grade level you teach.

If you teach younger children, ask a question focused on making sense of the numbers between 10 and 20. In order to get the most out of your students, it is a good idea to give a problem that has a context—this way if the student is not sure what to do, there may be some point of entry. For instance, a young child might be asked to work with the problem:

> There were nine eggs in a basket. The farmer collected 6 more eggs and put them in the basket. Now how many eggs are in the basket?

A 4th-grade student might be asked:

> The Murray family was driving 143 miles to Washington, DC. After 87 miles, they took a break, How many miles did they still need to drive?

A 6th-grade student might solve:

> How many packs of gum do I have if gum comes 14 sticks to a pack and I have 168 pieces of gum?

Examine the work you get from your students. Choose three students to write about: one whose mathematical work is sophisticated, and two whose work is not so sophisticated. Then write your analysis of these three students' work. For each student, your analysis should include:

a) What does the student understand?
b) What is the student missing that would enable more sophisticated mathematical work?
c) Based on what you see, what is a learning goal for the student?
d) What instructional strategies would support the student's learning goal?

Bring copies of your students' work to class. Be sure to remove or mark out the student's name. Label the papers Student A, Student B, and Student C.

Appendix B

Leadership II for Mathematics Specialists

Leadership II is the second in a series of three leadership courses for mathematics specialists. It is a 3-credit hour graduate mathematics education course designed to help prepare teachers with at least 3-years of classroom teaching experience to become school based K-8 mathematics specialists. The course will develop teachers' coaching skills related to work with adult learners and deepen their mathematics content and content pedagogical knowledge, as they refine their philosophy about mathematics teaching and learning. The course focuses on one-to-one coaching skills, the skills necessary to coach and lead small groups of teachers as well as providing school-wide professional development.

Leadership I, a prerequisite to this course, requires teachers to reflect on their own pedagogical and content knowledge while also studying research based "best practices" in mathematics instructional strategies. Leadership III which follows this course, allows the teachers to deepen and refine their skills as they learning to facilitate the Lesson Study process, create and use formative and summative assessments to diagnose student understandings and misunderstandings and learn to identify and use resources to address learning and teaching problems.

Course Description/Goals:

Leadership II course is designed for teachers to build those skills, understandings and dispositions required to play optimal, mathematics education leadership roles in elementary or middle schools. Prospective mathematics specialists who finish this course will:

- Develop and refine coaching skills and skills to work with adult learners.
- Build a deeper understanding of mathematics content and content pedagogy to develop standards based lesson plans.
- Develop planning and facilitation skills to lead small group and school wide professional development.
- Become familiar with the body of research related to selected topics within the National Council of Teachers of Mathematics (NCTM) strands in mathematics education.
- Build a deeper understanding of the mathematics that underpins the mathematics elementary and middle teachers will teach.
- Through personal reflection refine their philosophy about teaching and learning mathematics.

Course Overview

Leadership II is designed to follow Leadership I in which teachers reflected on their teaching in relation to current research on effective K-8 mathematics instruction. In Leadership I, the National Council of Teachers of Mathematics (NCTM) *Principals and Standards for School Mathematics* (PSSM) and *Common Core State Standards for Mathematics* (CCSS) are analyzed

with particular attention given to the Process Goals and the Mathematical Practices. In Leadership II teachers work to develop their coaching skills and study what it means to coach adult learners and become teachers of teachers. Assignments require the teachers to practice coaching one-on-one with another teacher in their school as well as working with grade level teams. They also continue to refine their own teaching practices and expand their knowledge of current research in mathematics education.

This course is highly interactive and project based. Each assignment is focused on the course goals. What follows is a brief description of each assignment supported by a rationale for requiring this work.

Pre Course Assignment

Participants will respond to a writing prompt provided by the instructors and read Chapter 1 from the West & Staub *Content-focused coaching: Transforming mathematics lessons.*

(Rationale: Students are asked to write a short paragraph describing their current thinking about what an instructional coach does or what coaching is. This provides a pre-assessment for the instructors as to what students are thinking about their role as a coach.. Focus questions on the West and Staub book as well as the Morse book help students to begin reflecting on how they can build their coaching skills.)

Journal—Due Each Class Meeting

Participants will make one journal entry each academic week between the first class and the fifth class in a 7 Saturday class format. Students will submit their journal entries at each class meeting. In addition, students will use these entries to develop a culminating reflection synthesis paper. The instructor will provide additional information and a rubric.

(Rationale: Journal writing supports our belief that reflection is the most significant form of growth and that the process of reflection required to make a quality journal is a significant activity. A student's journal provides a reference to highlight the continuum of their personal growth through the Mathematics Specialist coursework. Entries generally reflect an experience in their classroom, a discussion with a peer or some other thought that is relevant to the content and discussion from the course.)

Final Personal Growth Reflection Paper

Participants will develop a reflection synthesis paper using their journal entries to revisit the important ideas they have considered throughout the course. In their discussion, a student should highlight ideas related to their work with other teachers and ideas that they have considered about their own classroom instruction. The instructor will provide additional information about what to include in the portfolio and a rubric.

(Rationale: This culminating activity requires the student to reflect on the major themes of the course as they describe how a selected item represents a course goal and reflects their growth. They must explain why they selected that item to illustrate a change in their thinking. The course topics they must address include: Developing and Refining their Coaching skills, Building their capacity to Work with Adults in a Learning Community, Developing and Refining their abilities to Plan and Facilitate Professional Development

for Larger Groups, Refining their Philosophy of Teaching and Learning, Building a deeper Understanding the Mathematics for Teaching and Becoming familiar with the body of Research in Mathematics Teaching and Learning.)

Standards-based Lesson

Each individual will use the template provided by the instructor to develop a standards base lesson that focuses on classroom discourse, teach the lesson, reflect on the lesson, and analyze samples of student work from the lesson. The instructor will provide additional information and a rubric.

(Rationale: The goal for this project is to provide an opportunity for a prospective mathematics specialist to refine his/her knowledge and skills to create well-thought out standards-based lesson plans developed around a task or problem that calls up the mathematics content and process standards defined in the state and national standards, Van de Walle's three-part problem based lesson format, and Chapin's classroom discourse strategies. The lesson should be developed around a learning goal for one class period. The mathematics content in the learning goal should be well researched. The plan should reflect attention to how students develop understanding of the concept supporting the mathematics learning goal. The explanation should be of the quality of a leader in mathematics, that is it should be well thought out, clearly explained and with attention to details.)

Literature Review Project

Each participant will work in groups of 3 and each person will find 3 articles related to a specific topic in the elementary or middle school classroom. Topics include: algebraic reasoning with functions and change, connections between arithmetic and algebra, developing understanding and proficiency with basic facts, place value, understanding fractions as numbers, developing understanding and proficiency with adding and subtracting fractions, developing understanding and proficiency with multiplying and dividing fractions, proportional reasoning, geometry and measurement.

The components of the literature review project require:

Within the group and to the Instructors:

(1) Each group member to provide a copy of their annotated bibliography and the two-page summary of each article or book chapter. (A copy of each article or book chapter should be attached to the bibliography.)

(2) Each individual to develop a 2-3 page paper that synthesizes the key ideas to the annotated bibliography and addresses how these ideas inform his own teaching and work with other teachers. (Separate directions on writing an annotated biblography are provided.)

For the entire class

(3) The group will compile the annotated bibliography and send this electronically to everyone in class.

(4) Individual Synthesis paper

(Rationale: The goal for this assignment is to extend and deepen student understanding about a major topic in an important area of mathematics and to develop an understanding of the developmental continuum of the concept(s) and skill(s) within the

topic area. This assignment also introduces students to the knowledge and skills for locating, reading, comparing and contrasting, identifying the main ideas and synthesizing research. As a Mathematics Specialist or Teacher Leader they will be called upon to find research-based information for questions posed by school staff and parents, to learn about the impact of curriculum materials or particular instructional decisions, and to help inform school based policy decisions about the mathematics program. Students are required to use the APA format to gain experience in professionally writing and documenting their work. This project provides valuable experience prior to the Mathematics Education Research course students will take the last year of their graduate program.)

Coaching Project

For this assignment, participants will plan and videotape a coaching session (one pre-conference and one post-conference) with an identified classroom teacher. As part of this assignment, students will choose one 10-minute uninterrupted clip to present to a small group of classmates. After discussing the clip with the small group, students will develop a two-three page written summary of the important ideas related to mathematical pedagogy and content that surfaced during their own pre- and post-conferences, and a critique of their skill as a mathematics coach. (Additional information and a rubric are shared by the instructor on the coaching project requirements.)

(Rationale: The purpose of this assignment is to provide an opportunity for prospective Mathematics Specialists to develop the knowledge and skills about content focused coaching to support and provide professional development opportunities for mathematics teachers. As participants complete a coaching cycle with a classroom teacher, they are asked to reflect on the thinking that was involved in planning the preconference and post conference in terms of their own thinking about the mathematics topic of the lesson and why they developed the particular questions they posed; the important points that came up during the lesson and connections between what was said in the preconference and observed in the lesson; how well the mathematics and mathematics pedagogy were developed during the pre-conference; analysis of three students' work; what important features were brought out in the post-conference conversation and why; ideas for refining the lesson; next instructional steps and what they would do differently; the rationale from why they choose a particular segment of the video to share; how the experience will affect them as a coach or teacher leader; what did they learn about working with another adult to plan a lesson and what do they need to learn more about to further their development as a coach. Responses to these questions require a metacognitive approach which will be a valuable practice for participants to utilize in their work.)

Planning and Facilitating Case-based Professional Development

In groups of 2-3 participants will use the template provided by the instructor to plan and facilitate a 1.5-hour professional development opportunity in class based on a chapter assigned from Amy Morse's *Cultivating a Math Coaching Practice*

(Rationale: Cultivating a Math Coaching Practice serves as a tool that allows mathematics specialists and teacher leaders the opportunity to gain experience as a

facilitator of professional development and to reflect upon real coaching images such as teachers working one-on-one with a coach, coaches providing professional development, coaches working on their own practice and the decision-making that takes place within that practice. With the other class members as their audience, participants plan, implement and reflect on their assigned topic (chapter) related to the work of a mathematics coach. Presenters must think about the focus questions they will ask in small group and whole group and how these relate to their overall goals for the session. In reflecting on the lesson they are asked to share what they learned about designing and facilitating professional learning opportunities; what they learned about collaborating with a team to develop and implement professional development and what they identify as their own strengths and needs as a facilitator.)

Planning a Workshop

Students will work in pairs to develop a 3-hour workshop designed for the faculty in an entire school. The workshop will be interactive and the plan will include clearly defined goals related to the participants learning, an annotated agenda for the three hours, at least one math activity, at least one reading, how the participants learning will be evaluated at the conclusion of the workshop, and a reference list indicating the research support for the activities in the workshop. Additional information and a rubric will be provided by the instructors.

(Rationale: This assignment has two components. The first goal involves collaboration with the Principal and a reflection paper about this meeting. The meeting allows the participant to get to better know their administrator as the school's mathematics education leader and to discuss the school's mathematics program. Sample questions include: What does the administrator see as the strengths and weaknesses of the school's instructional program? What professional development experiences does the administrator believe would be helpful for the staff? Based on your conversation what do you perceive are the administrator's beliefs and attitudes for learning mathematics? The second component is planning a 3-hour workshop for their staff based on needs reflected in the principal conversation. The focus must have a strong mathematics component. They must conduct a related literature review to determine what the experts have determined on their topic. Then they must set goals for the workshop and design the workshop. A planning and annotate agenda template is provided to guide their work as most participants are just learning what it means to be a "teacher of teachers" vs a "teacher of students". Required in the plan is at least one mathematics task. The mathematical concepts and skills of the task must be articulated and multiple strategies for solving the task must be noted. The strategies must be connected and notes must indicate where one might expect confusions or misconceptions. Desired representations and necessary tools must be listed along with expectations of how participants will work together but also provide individual accountability. They must address how participants will explain and justify their solutions and how you will know they are learning. There should also be a description of how the task will be launched. The plan should be organized in a format that gives the time allocated, the learning activity for each time slot, the facilitator's moves and/or questions and expected reactions and results and any materials and supplies that are needed. The refection paper on this second component of

the project should speak to the decision making process to lead them to their topic and goals, discuss the research that lead to the workshop, explain how the activities will move the participant toward the goals, some idea about how you might use this design in the future and finally how they have grown and still need to grow in this process.
Note that this project is only the planning for a professional development session. The logistics of actually providing a workshop can be problematic as most of the participants are still classroom teachers and formal professional development time within a school is limited. It maybe that after the PD is planned and shared with the principal that time is allotted for the session but this is entirely optional.)

Course Format and Key Activities

This course has been provided in a variety of formats. In a regular semester, class is held for 15 three-hour sessions. Other options have been to offer 3 to 4 six-hour sessions in the summer followed by 4 to 5 six-hour sessions held about once a month on Saturdays or to hold 7 Saturday class session over the course of a school year. Each timeframe impacts the participants in different ways. During a semester class, the teacher participants are generally working full time and have opportunity to connect their learning to their current school environment throughout the course. At the same time this format does not allow the participants much time to reflect on each assignment or collaborate extensively with others. When the course begins in the summer and continues into the school year, there is more time for reflection and preparation of each assignment. Instructors however must design and assign projects appropriately. For example, implementing a lesson plan would need to be due when participants have access to a class or students. A literature review, on the other hand, could be assigned in the summer.

Regardless of the format, the course design is student-centered and inquiry based. Instructors strive to model the instructional strategies that participants are expected to implement and share with their teachers as they become Mathematics Specialists. Discussions often begin with individual reflection, followed by pair conversations, expanding to small group and then whole group sharing. At times the instructors "step-out" of their teaching role and lead an explicit discussion about the teaching moves they just employed. Being able to identify and articulate these strategies is an important coaching skill these future specialists will need.

Though the course goals focus on building leadership skills, doing mathematics together is also an important component. Activities include doing mathematics and mathematical tasks, looking at student work and considering teacher moves. Written and video cases on teaching and coaching scenarios primarily come from the recommended texts but other resources are introduced as necessary to meet participant needs.

The assigned projects are designed to develop participant content knowledge and leadership skills as well as their communication skills. Many opportunities are provided for participants to articulate their ideas both in writing and orally, individually and in a group. This ongoing formative assessment allows the instructors to monitor and guide student development and allows the participants ways to access their own leadership abilities and understanding of the complex nature of adult learners and the demanding role of a mathematics specialist.

Course Materials
Listed below are the primary student and instructor texts for the course. In addition, instructors will include supplementary readings such as journal articles.

The Primary Student Texts

Hansen, Pia M. (2009). *Mathematics coaching handbook: Working with teachers to improve instruction.* Eye on Education Press. Larchmont, NY. ISBN 978-1-59667-093-8.

Morse, Amy (2009). *Cultivating a coaching practice: A guide for K-8 educators*. Corwin Press. Newton, MA. ISBN 9781412971065

West, L. & Staub, F. C. (2003). *Content-focused coaching: Transforming mathematics lessons*. Portsmouth, NH: Heinemann. ISBN: 9780325004624.

Instructor Primary Resources:
National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (2000). *NCTM Principals and Standards for School Mathematics* Reston, VA: Author.

National Council of Teachers of Mathematics (1991). *NCTM Professional Standards for Teaching Mathematics* Reston, VA: Author.

Kilpatrick & Swafford, eds (2002). *Helping children learn mathematics,* National Research Council, Washington D.C.

Costa, A. & Gamoran, R. (2004). Cognitive *Coaching: A foundation for renaissance school.* Christopher Gordon Publishers.

Sowder, J., Schappelle, B. (2002). *Lessons learned from research*. Reston, VA: National Council of Teachers of Mathematics.

Van de Walle, J. Karp, K. and Bay-Williams, J.M. (2010). (7^{th} edition). *Elementary and middle school mathematics: Teaching developmentally*. New York, NY: Pearson Education (Allyn & Bacon). (A student text in Leadership I)

Supplemental Reading Suggestions
Carroll, C., & Mumme, J. (2007). *Learning to lead mathematics professional development.* Corwin Press.

Clements, D.H. (1999). "Concrete" manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*(1), 45-60

Chapin, S. H., O'Connor, C., O'Connor, M. C., & Anderson, N. C. (2009).Classroom discussions: Using math talk to help students learn, Grades K-6. Math Solutions.

Killion, J. (2010). Reprising coaching heavy and coaching light. *Learning Forward*, *6*(4), 8-9.

Lipton, L., & Wellman, B. (2007). How to talk so teachers listen. *Educational leadership*, Association for Supervision and Curriculum Development

Merritt, E., Rimm-Kaufman, S., Berry, R., Walkowiak, T. & McCracken, E. (2010) . A Reflection framework for teaching math (MSCANS). *Teaching Children Mathematics.,17*(4), 238. Retrieved from http://www.nctm.org/eresources/view_media.asp?article_id=9507.

National Staff Development Council (2006) . NSDC tool: Developing a partnership agreement between a coach and a principal 2(4), 8-9.

Course Outline: Topics and Essential Questions

Assignments: *(Instructor note: In addition to a detailed narrative description of each assignment, it is important to provide a rubric for grading and whenever possible a sample paper to serve as an example for students. This will help avoid confusion over expectations and help set a clear standard for student work.)*

Textbooks:

MCH: *Mathematics coaching handbook: Working with teachers to improve instruction.*
CMCP: *Cultivating a coaching practice: A guide for K-8 educators*
CFC: *Content-focused coaching: Transforming mathematics lessons*.

Each class is 6 hours

Class	**Topics/Resources**	**Essential Questions**
1	Working with Adults/ North, South, East, West activity (from National School Reform Foundation) Establish class norms Multiple roles of a mathematics specialist Purposeful planning and facilitation of team meetings	What are the skills and knowledge necessary to work with adults? What should be our class norms? What are the diverse roles of a mathematics specialist? What is necessary to develop a workshop using commercial

	Project Review: Facilitation of Chapter in Morse book, Standards-Based Learning Lesson Planning and Reflective Journal MCH: pages 1-12 CCFC: The Foreword, the Acknowledgements, the Preface, Chapter 1 CMCP: pages 57-78, 161-164	curriculum materials? What are the potentials of grade-level meetings? How can we strategize to create meaningful agendas? What are ways to balance sharing responsibility for the agenda and facilitation of meetings?
2	Participants facilitate CMCP, Case 9, "Taking the Lead as a Teacher of Teachers" Develop understanding of Content Focused Coaching, CFC Chapter 1 (3 phases of coaching cycle), chapter 2 (model for working with one teacher- but could apply to groups) chapter 6 (coaching the experienced teacher) Introduce the Coaching Project Review the Standards-based lesson planning project MCH: Chapter 2 CFC: Chapter 1,2,6 CMCP: Chapter 9	What does it mean to be a teacher of teachers? How does one refine their knowledge and skills for coaching and build a climate for coaching? What is content focused coaching? As a coach, how do you get into classrooms? As a coach, how do you have conversations with another adult about their practice? As a coach, how do you maintain a stance of inquiry and support and not judgment?
3	Participants facilitate CMCP, Case 1:	What is involved in preparing to

	"Observing Studying, Analyzing Planning: Preparing to Coach" Challenges faced by coaches and teachers in negotiating relationships Coaching cycle and how coaches facilitate teachers work in each cycle Participants facilitate CMCP Case 2: "Discerning and Responding: Coaching in Real Time" Knowledge and skills to read and synthesize research articles. Discuss Literature Review Project MCH: Chapter 4, pages 37-51 CFC: Chapter 5 CMCP: Chapters 1 and 2 Clements (1999) article	coach? What is involved in coaching a new teacher? What does it mean to coach in real time? What knowledge and skills are important in reading and synthesizing research articles?
4	Participants facilitate CMCP, Case 3: "Strategic Coaching: Goal-Centered Modeling in the Classroom" Continue to develop knowledge and skills for coaching. Explore the various stances that a coach takes when working with teachers at various levels of expertise. Continue to develop the knowledge and skills as a mathematics specialist to facilitate teachers in writing a Standards	What does it mean to coach strategically and how is it done? What are the various coaching stances and what can be gained or lost by coaches assuming various stances in a coaching situation? What is conceptual knowledge and what is procedural knowledge? How can a framework that measures student engagement be used to facilitate a conversation about a lesson

	Based Lesson. Become familiar with the MSCAN framework to examine student engagement. Develop the knowledge and skills as a mathematics specialist to conduct a literature review on a mathematics topic and to develop a written document based on the review. CFC: Chapter 5 (post-conference video) CMCP: Chapters 3 and 4 Clements (1999) article reread "Continuum of Coaching Behaviors" Ellis (1998) "A Reflection Framework for Teaching Math (MSCAN)" Merritt et al (2010)	and how it engages students? How can a literature review best be developed and shared?
5	Explore the decision making process during professional development facilitation and reflect on how the purpose of the professional development impacts the decision making. Refine coaching knowledge and skills by engaging in simulation "coaching the coach" through the sharing of a coaching video. Explore issues and challenges that arise when working with administrators and how to cultivate effective working relationships with administrators.	How does the purpose of professional development impact the facilitator's decision making? What are sociomathematical norms and what implications might they have for mathematics specialist facilitating professional development? How can the coaching cycle be helpful to the work of a mathematics specialist? What is the importance of establishing a relationship and job expectations with the principal?

	Project Guidelines to Review: Planning a School wide Workshop and Administrator's Conversation CFC: Chapter 8 Carroll, C. & Mumme, J (2008) Resources and Video Lipton and Wellman (2007) article NSDC tool (2006)	
6	Participants facilitate CMCP, Case 12: "Examining the Role of Authority" Refine skills and knowledge for implementing and fulfilling the role of a mathematics specialist. Debriefing the Literature Review Understanding and leading change Project Guidelines to Review: Final Personal Growth Reflection Paper MCH: Chapter 2, 3 and 6 CMCP: Chapter 12 Killion (2010) article	What is the role of authority as it relates to school based mathematics coaching? What does author Killion suggest constitutes coaching heavy vs coaching light? What does this mean for mathematics specialists work with teachers? In what ways is a mathematics specialists a "change agent" in their school?
7	Participants facilitate CMCP, Case 11: "Framing the Connection Between Coach and Teacher Goals" Building relationships with teachers	What strategies can a mathematics coach implement to frame a connection between their goals and teacher goals? When are different types of coaching

	around their classroom practice Further refine ones knowledge and skill to design and facilitate mathematics content focused workshops. Refine ones skills and knowledge for coaching in diverse situations Reflecting across the leadership courses and making explicit the skills, characteristics, resources that will help them DO the job of a specialist "Continuum of Coaching Behaviors" Ellis (1998) MCH: Chapter 8, 93-104	behaviors most appropriate? What are the advantages and disadvantages for each type of coaching? What strategies might help address various school scenarios in the short term and long term? What skills, characteristics and resources will help them do the job of a mathematics specialist?

Sample Lesson Plan for Class 3: Preparing to Coach

Textbooks/Resources:
Morse, Amy (2009). *Cultivating a coaching practice: A guide for K-8 educators*. Corwin Press. Newton, MA. ISBN 9781412971065

West, L. & Staub, F. C. (2003). *Content-focused coaching: Transforming mathematics lessons*. Portsmouth, NH: Heinemann. ISBN: 9780325004624.

Essential Questions:
What is involved in preparing to coach?
What is involved in coaching a new teacher?
What does it mean to coach in real time?
What knowledge and skills are important in reading and synthesizing research articles?

Welcome and Logistics (10 minutes)

- Review posted agenda
- Collect Journal Entries
  - Entry 1: Read Chapter 4 in Hansen book, pages 37-51. Write a 1-2 page double spaced reflection that discusses what you see as the key ideas in this chapter for mathematics specialist. What are some strengths that you bring that will help you and

where do you see that you will need to cultivate strengths to enhance your work as a mathematics specialist?
    - Entry 2: In 1-2 pages think about your own teaching and how working with a coach could help you grow. How might you reach out to a coach for this help?
- Collect the Standards Based Lesson Plan Project: time will be spent discussing these in Class 4
- Homework from Class 2 to prepare for Class 3:
    - Guidelines for the Coaching Project were shared. Students were to review these prior to Class 3 and begin thinking about what teacher they want to invite to work with them on the coaching project.
    - Guidelines for the Standards-based lesson plan were shared. This was due on Class 3 and the short turnaround time was by design. As mathematics specialists they will need to be able to think through and develop well thought out lessons quickly.
    - The groups facilitating Chapter 1 and 2 from the Morse book will present during Class 3

Group Presentation (1.5 hours)

A group will present Morse's CMCP Chapter 1: "Observing, Studying, Analyzing, and Planning: Preparing to Coach." (Facilitating a chapter from Morse's book was modeled in Class 2 with the instructors "stepping in and out" to make explicit there decisions in using this prepared resource.) Groups have been assigned various chapters to facilitate, this being the first. The other students act as professional development participants and give feedback as "Grows and Glows". The instructors will use the shared rubric to score the presenters.

West Content Focused Coaching (1.75 hours)

Chapters 1 and 2 of the Lucy West book, Content Focused Coaching, were assigned for homework after the first class. The discussion here will begin with a review of the 3 steps in the coaching cycle. In Class 3 students will explore Chapter 5 on coaching the new teacher and how it differs from coaching a veteran teacher.

- Provide the first page of a 3 page handout will be shared and teachers will follow the directions, first reading pages 48-49 and thinking about the following:
    - What do you know about the teacher? (background, knowledge of mathematics, inquiry level)
    - What do you know about the class of students? (grade level, beliefs and attitudes about mathematics)

- Work in table groups to "Do the Math" and consider the big ideas and key mathematics concepts:

> ***You have eleven fruits in your basket. Some are one kind of fruit and the rest are another kind. How many of each could you have?***

- Discuss the following questions: As K/1 students work on this problem what representations do you think they may use? What might they notice? What combinations are they likely to find?
- Hand out Page 2 with has guiding questions for viewing the video of a lesson (40 min) with a new teacher. There is a column for notes about the lesson and a column for notes about how they would address this issue in a post conference. Teachers also have time to review the transcript for more details.
- Distribute Page 3 which has participants watch the preconference (40 min) with helpful suggestions for focusing their review and comments.
- Lead a whole group discussion around the handout questions.
- Share that the homework journal entry will be to watch the post conference.

Clarify Guidelines for Coaching Project:
Review the directions for the Coaching Project highlighting the following general directions about what the activity includes:

- Planning, conducting, and video-taping a pre-conference and a post-conference with a teacher around a lesson. The pre and post-conference should be no more than 20 minutes each.
- Collecting data on student engagement and learning during a classroom observation of the lesson being taught. The actual lesson that is taught by the classroom teacher does not need to be videotaped.
- Analyzing the pre and post-conference video along with any preparation notes you made, the lesson plan (this may be from the textbook), samples of 3 student's work, and any other data collected during the observation or the pre/post conferences.
- Sharing a 10-minute clip of the pre-conference or post conference with peers in class. Choose one 10-minute uninterrupted clip of the pre-conference to share with a small group of classmates and to serve as a reference point for talking about your coaching experience.
- Writing a reflection paper about the coaching experience.

Hand out the detailed description along with the scoring rubric.

Read seminal article by Doug Clements, "Concrete" manipulatives, concrete ideas. Contemporary Issues in Childhood Education 1 (1), 45-60. (15 minutes)

Reading the Research (30 to 45 minutes)

- Facilitate a discussion on the guidelines for reading research articles. Handout list of suggestions.

- Explain what a seminal article is and how it can be used in a literature review. Along with providing some ideas for how to gain skill for reading scholarly articles.
- Table groups discuss questions on the handout (15 min)
- Ask each table to take one question and share their insights. Ask: What did they notice about the format of the paper the writing style, etc.
- Share the guidelines for the Lit Review Project and have participants sign up for topics from the suggested list.
- Ask participants to reread the article for homework and respond to handout of focus questions.

Clarify Questions on the Lit Review Project (10 minutes)

Homework (25 minutes)

- **Journal Entry:**
  Review the directions for the next journal entry, due Class 4, provided on a handout.
  - Entry 1: Watch the CFC Chapter 5 (New Teacher Sillman) Post-Conference video. Write a 1-2 page double spaced reflection on the post-conference. Speculate on what mathematics the teacher may have learned that is specific to this lesson, and to the practice of teaching and learning in general, as a result of these coaching segments. Provide support for your speculations. In addition reflect on the following:
    - How does the coach help the teacher foster student learning in the lesson coached?
    - How does the coach help the teacher develop teaching expertise in general?
    - In what ways does the coach address core questions and issues in lesson design and analysis?
    - How does the coach use information from the pre-conference and lesson observation to shape the post-conference?
  - Entry 2: Free write. Pick one or two ideas that have come up during the first three classes and in 1-2 pages double spaced reflect on how these ideas have helped you to expand your thinking and/or helped you to refine your teaching practice.
- **<u>Remind the group of the Projects Underway</u>**
  - Coaching Project
  - Literature Review Project

- **<u>Amy Morse Cultivation a Coaching Practice Presentation Project</u>**
  Remind the group that the next two groups will present during Class 4.
  - Chapter 3: Strategic Coaching: Goal Centered Modeling in the Classroom
  - Chapter 4: Reaching a new Teacher: Math as the Conduit

- **Article Reread:**

Ask students to reread the Clements article and use the handout “Reading Research Based Articles: Table Group Discussions” to take notes and prepare for discussion at the next class meeting.